SPECIAL CONTRIBUTIONS.

STUDIES ON THE STATICS AND KINEMATICS OF THE ATMOSPHERE IN THE UNITED STATES.

V. RELATIONS BETWEEN THE GENERAL CIRCULATION AND THE CYCLONES AND ANTICYCLONES.

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UNEQUAL DISTRIBUTION OF CYCLONES IN NORTH AMERICA AND EUROPE-ASIA.

We have arrived at the following proposition as the result of the discussion of Ferrel's and Oberbeck's analysis of the general and local circulation, that the general cyclone and the local cyclones and anticyclones have been treated almost independently of one another, while in fact the imperfect results of the theory and the modern observations both indicate that these two classes of movement should be analyzed in close relation with each other. The evidence compels us to regard both these circulations as the common effect of the readjustment of the thermal equilibrium, which is disturbed by the radiation of the sun falling on the tropic zones, and the true meteorological problem is to trace out the successive stages in the process of this interaction through the resulting currents which circulate in the atmosphere. The results contained in this paper apply especially to the North American Continent, and it is hardly to be expected that the details will be found the same in all the other regions of the earth. Indeed there are several reasons for believing that this continent is the peculiar theater for the interchange between the heat and the cold of the Northern Hemisphere, and that the Euro-Asian Continent plays a very different role in the meteorological economy of this hemisphere. For it is well known (1) that while the American Continent is the place for the profuse generation of cyclones, Europe is the region for their dissipation, and in Asia very few cyclones occur except along the ocean borders; (2) that the velocity of motion of the atmosphere generally is about twice as great over North America as over Europe. This points to a profound difference between the actions of the atmosphere in these two regions, but one cause of it at least is easily perceived. It has been shown that the currents which are especially concerned in forming cyclones are contained for the most part within 2 or 3 miles of the ground, though their accompanying effects may extend much higher. Hence, any barriers of elevated ground, as mountain ranges, which tend to deflect the flow of the lower strata, must strongly influence the formation of the cyclones themselves, if they are to be referred to the counter flow of long horizontal currents of different temperature rather than to local vertical convective currents.

The great range of the Himalaya Mountains stretching east and west is such a barrier to the flow of the tropical and polar currents in that region, and the result is that true cyclonic movements are almost excluded from the interior of Asia. On the other hand the Rocky Mountain range, stretching north and south along the western districts of North America, favors the counter flow from the Tropics and the polar regions by deflecting the westward current of the Tropics toward the north, and the eastward drift of the higher latitudes toward the south. The same tendency is favored by the location of the high pressure belt in the latitude of 35°, which causes a high pressure area to form over the middle Atlantic Ocean, while the Rocky Mountain range breaks through the midst of it. The result is to produce a powerful anticyclonic center of action over the Atlantic Ocean, which produces a strong northward component from the West Indies toward the interior of the continent. At the same time the American Continent causes the isobars and isotherms to loop southward, and thus in consequence to draw the Siberian atmosphere in a direction nearly parallel to the Rocky Mountain range. These physical conditions are a constant incentive to the formation of countercurrents which meet on the Canadian and United States territory, with the result that 75 per cent or 80 per cent of the storms of the Northern Hemisphere are generated in these districts. It is not necessary for maintaining the temperature equilibrium of the hemisphere that the interchange of heat and cold should occur so as to have a uniform distribution over all portions of it, because if there is an excessive interchange in any place, as in North America, the general movements of the atmosphere will soon transfer the effects to all other parts. Keeping these facts in mind will facilitate an understanding of the views which will be briefly described as follows:

CRITICISM OF THE CANAL THEORY OF THE GENERAL CIRCULATION.

The immediate problem before us is this: To what extent is the canal theory of the general circulation over a hemisphere correct, and in what direction must it be modified to conform to the modern observations? Ferrel derived the following equations from a discussion of the first equation of 397a for the approximate velocities and gradients in the strata of the upper atmosphere:

$$408a. \quad \text{Velocity;} \quad v = v_0 - \frac{0.016}{(2n+\nu)} \cdot \frac{gh}{r} \cdot \frac{A_2 \sin \theta}{(1+at)};$$

$$409a. \quad \text{Gradient;} \quad G = G_0 - \frac{0.00001327}{\cos^2 i} \cdot \frac{A_2 \sin 2\theta}{(1+at)^2} \cdot \frac{P}{P_0} h;$$

when v_0 G_0 P_0 are the values at the surface, and v, G, P, the values at the height h. Since A_2 is negative, — 20.95°, it follows that v is greater than v_0, and G is greater than G_0, so that the eastward velocity and the meridional gradient velocity increase with the height. The relative eastward velocity is shown on fig. 16. The interchanging velocity along the meridians is northward above the neutral plane and southward below it as given on fig. 16.

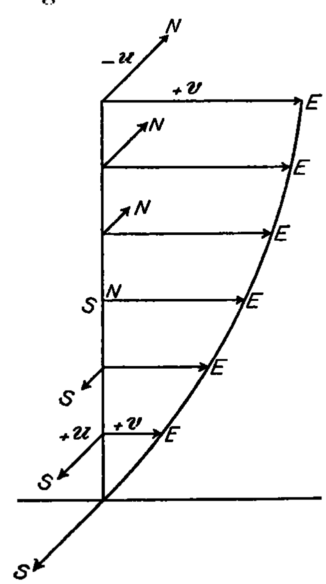

FIG. 16.—Ferrel's component currents by the canal theory.

The canal theory requires that the flow of the atmosphere should be in unbroken stream lines conforming to these precepts. The result of the principle of preservation of areas, $v\varpi =$ constant, as applied to the axis of rotation of the earth, is that the velocity v is excessive near the poles, and the gradients of the upper strata much too large. Ferrel sought to escape this difficulty by evaluating $\Sigma m\ (v - v_0)\ k$ for the upper strata with a northward $- u$, and $\Sigma m\ (v - v_0)\ k$ for the lower strata with a southward $+u$, and assuming that the difference of momenta is equal to the retardation of the eastward drift by the frictional resistance. It is known, however, that the frictional coefficient is a very small factor, and not capable of producing the required retardation.

Furthermore, the modern observations show, as in Paper III,[1] that the northward $-u$, and the southward $+u$, components are not distributed as the canal theory requires, but that there are approximately equal currents flowing northward and southward *on the same levels*, which reduces the difference of momenta to zero, and is fatal to the frictional theory of retardation. The counter flow of horizontal currents on the same levels, most powerful in the strato-cumulus level, does, however, constitute a dynamic mechanism quite capable of retarding the eastward drift, so as to produce the observed moderate eastward velocities of the temperate zones. This is the true source of the energy consumed in the motions of cyclones and anticyclones. Compare fig. 17.

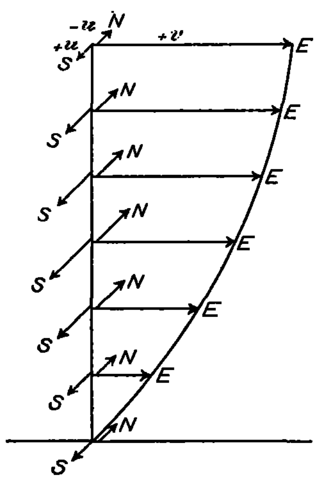

FIG. 17.—Bigelow's component currents from the Weather Bureau observations.

The general theory which has already been mentioned in my previous paper—Storms, Storm Tracks, and Weather Forecasting, Bulletin No. 20, Weather Bureau, 1897—is practically as follows: There is a sheet of the atmosphere flowing quite steadily eastward over the United States at the levels from 2 miles upward. Beneath this flows north and south, quite independently of the upper drift, a series of countercurrents which are cold and warm, respectively. The interaction of these currents sets up the anticyclonic and cyclonic whirls chiefly by dynamic action, the former causing a downward component and the latter an upward component. The cyclone receives its supply from the northwest and from the south, and thus discharges great masses of air through itself, so that the cyclonic configuration constitutes a type of persistent circulation in a vertical form of stream lines. This gyratory rotation lifts the air from one level to another, purely by its mechanical action, and thus raises the mass from the strata of lower to those of higher eastward velocity. This rapid change of eastward motion, if imparted to the air raised up, can take place only by an interchange of inertia, that which is gained by the rising air being lost by the upper eastward drift. Also the gyration of the lower strata causes deflection in the eastward drift by the composition of forces, as must be the case, since the method followed in our discussion has been to subtract the normal west-east and north-south velocities from those actually observed, the residuals being the circular cyclonic movements that are now described.

The point of view for the consideration of the theory of the general cyclone is thus considerably altered. (1) Instead of a circulation as in a canal, northward above and southward below, we find that the interchanging motion is largely confined to the lower strata, by means of currents not flowing above one another, but on the same level. (2) Instead of the momentum $m\,(v_0 - v)$ being determined by the difference between the eastward flow at different levels of the same latitude, we prefer the statement that cyclonic gyrations are produced by the *counter flow of independent streams, and that the rapid* eastward drift is retarded by mechanical inflows, from above toward the base of the anticyclone, and from below through the stream lines of the cyclone into the eastward drift. The energy upon which cyclones and anticyclones depend for their activity is to be traced to a different source from that generally assigned to it by meteorologists. The common theory is that the cyclone is due to some form of vertical convection, caused by overheating a local region, and by the latent heat produced in precipitation. Our theory would more naturally depend upon horizontal convection, by means of which temperature gradients thousands of miles in extent produce comparatively shallow streams, which flow from the north and from the south, and sustain them for considerable intervals of time. The upward and the downward discharges, together with the rotary components which make the sinuous flow of the air in the upper levels, practically tie together the upper and the lower strata, retard the eastward drift in the higher strata, and accelerate the eastward motion in the lower. This effect is readily perceived in the eastward propagation of high and low pressure areas over the United States, which is the basis of our system of forecasting and renders it possible.

It is by no means to be concluded that by suggesting this modification of the Ferrel theory, any intention exists of not recognizing fully the fact that it remains substantially correct in some of its features. There exists the eastward drift throughout the middle latitudes and the westward drift in lower levels of the tropical zone. But there is yet another reservation to be introduced. The heating of the tropical belt raises the isobars adjacent to the equator so that they slope toward the poles, and to such an extent that they almost exactly counterbalance the deflective force $2nv \cos \theta$, which is directed southward. It may properly be assumed from this that in the upper strata the directions of the isobars, the isotherms, and the stream lines of the wind motion are very nearly parallel to each other, if not coincident. The friction is evidently small, judging from theoretical conditions, and from the results of the observations, or else this could not be the case.

There is another important deduction to be drawn from this discussion regarding the flow of currents from the Tropics toward the poles in the lower strata. According to the Ferrel theory, the overheating of the Tropics is relieved by the upward expansion and overflow, but in accordance with the present view the tropical congestion is relieved by irregular streams which flow outward from the lower levels. This being the case, the poleward gradients in the upper levels are called upon to sustain much less pressure from the deflection, and evidently the tendency to excessive eastward velocities is much diminished, just because the equatorial lift of the strata can not be so great, since the expansion upward, as stated, leaks off sidewise in the lower levels. The eastward drift does not therefore increase to excessive values for these two reasons: (1) The tropical strata are not elevated up to the theoretical amount because of the escape of the currents poleward not very far from the surface of the ground; (2) the eastward drift is diminished by the operation of the vertical discharges between the lower and the higher levels produced by the purely mechanical vortex motion in the cyclones and the anticyclones.

The evidence before us is to the effect that the heating of the atmosphere is generally confined to a layer less than 5,000 meters, or 3 miles, thick. It is not intended to allude now to

[1] MONTHLY WEATHER REVIEW, March 1902, Vol. XXX, p. 117.

the annual range in temperature with the sun's change in latitude, but rather to the shorter periods of only a few days length which contribute to the impulse of streams from the south. There are several reasons which lead to this conclusion: (1) The trend of the preceding argument has been to show that the readjustments of disturbed temperature equilibrium take place in the lower layers of the atmosphere by means of rather spasmodic impulses, controlled partly by the temperature energy in the tropic and the polar regions, partly by the distribution of land and ocean temperatures, and by the relative radiation which takes place from them in the winter and summer season, respectively. (2) Half of the mass of the atmosphere is contained below the 5,000-meter level, and this is the layer within which the greater part of the aqueous vapor is also collected, since the vapor contents of the higher levels is in the form of fine ice crystals, which drift eastward, encircling the earth, and perhaps seldom finding their way to the ground. The fact that the dust of the Krakatoa volcano was thus carried about the earth for two or three years shows that the upper current has a history of its own, to a considerable extent independent of the 3-mile layer nearest the earth. Now, the important part which the aqueous vapor plays in the absorption of heat is well understood, and it depends upon the very high latent heat of water, which is 606.5 calories per kilogram at 0° C. The evaporation and condensation of water in precipitation is certainly confined to the lower stratum, and hence it is in harmony with this view to limit the effective heating of the air by the sun to the lowest 3 miles. (3) The diurnal variation of the temperature at the surface of the ground takes on a wide range. The temperature is above the normal in low areas, in the summer season, and in the middle of the day; it is below the normal in high areas, especially in the case of cold waves in the winter, and in the night-time. The range at the ground generally amounts to 10° or 20° F., but diminishes upward with the height and disappears at the 5,000-foot level, or even considerably below that height. This is shown very clearly in the study of the changes in the vertical temperatures, as explored by means of balloon ascensions, where the 5,000-foot level marks the convergence of the lines which represent the gradients in the forenoon and afternoon. The range in the United States is usually greater than in Europe at the ground, owing to the more pronounced nature of the cold and warm waves that move eastward over that region, but the evidence is that the diurnal lines converge at 2 or 3 miles above the ground. (4) The fact that the great eastward drift of the upper levels is underrun by a series of comparatively thin currents, which move about in every possible direction, shows that the disturbance of equilibrium is local and confined to a shallow skin near the ground, the most rapid currents belonging to the cumulus and strato-cumulus levels, which also implies that the upper regions of the atmosphere are much less affected than the lower. (5) The same conclusion is indicated very clearly by the seasonal change in the drift of the high areas over the northwestern portions of the United States, from the northwest in winter and from the southwest in summer, in conformity with the location of the permanent high areas in winter over the continent and in summer over the ocean.

From these considerations it seems evident that the upper atmosphere is but slightly disturbed in its temperature equilibrium by the effects of the solar radiation, but the solar rays pass through it with comparatively little absorption, while the larger percentage of the heat retained in the lower strata, is due to a change of the wave length. This conclusion is very important for two reasons: (1) It shows why the general circulation of the atmosphere prescribed by the Ferrel-Oberbeck theory does not seem to be confirmed by the observations. (2) It also indicates where the energy comes from which is finally expended in the generation of cyclonic circulations and in the retardation of the eastward drift by the agency of inertia rather than by friction; for if the total amount of energy falling upon the Tropics does not expend itself in an upper poleward current, because the higher strata retain a temperature of equilibrium almost undisturbed, then this energy must give rise to a series of comparatively small currents moving poleward in the lower strata—a fact which is abundantly confirmed by the observations. Also, if the friction of the upper atmosphere upon itself is very slight, then there will be but little retardation to an excessive eastward drift, tending under a steady force to have a constant acceleration. The only other available agency which will produce the same effect is the intrusion into the higher strata from the lower, or vice versa, of air moving eastward with a different velocity, which must suddenly be subject to acceleration. The discharge of the product passing through cyclonic circulations is perfectly fitted to perform this office. Hence, the theory here expounded consists of two parts as regards the eastward drift: (1) The upper poleward gradients are not built up to the amounts supposed by Ferrel, because of the lateral escape poleward of currents from the tropical belt; (2) The agency of friction as a retarder is replaced by the interchange of inertia derived from a compound circulation, the sources of the separate parts having different and independent causes. It has been important to thus carefully clear the ground for the theory of local cyclones, which will be advanced to take the place of the type proposed by Professor Ferrel on the one hand, or of that advocated by the German school on the other hand. These two theories are not in harmony with each other, and neither of them seems to be in agreement with the observations. It is very evident that the six assumptions which have been made in order to pass from the general equations of motion, 200, to the working system employed by Ferrel, 397a, must be carefully revised before we can expect to put this branch of meteorology upon a correct working basis. In particular it is not suitable to omit the variations of temperature in longitude, because in so doing the turbulence of the lower strata and the alternate streams which are implied in this variation profoundly modify the general and the local circulation. It is also necessary to correct the statement regarding the friction as the special agency of retarding accelerated flows and substitute, or rather add thereto, the inertia of currents which are rapidly changing their velocities. The conflict of turbulent countercurrents, especially at a short distance above the ground, must be rigorously considered in studying the resultant effects. Likewise the direct application of the law of conservation of areas passed over by the rotating radius vector can not apply immediately to the lower strata, though it may be much more nearly correct above the 3-mile level.

MODIFICATION OF THE CANAL THEORY.

In consequence of these considerations it seems necessary to modify the canal theory to such an extent as to be practically equivalent to an abandonment of it. If this is done it is important to trace out a chain of circumstances which will give a more correct account of the general and the local circulation of the atmosphere. The canal theory is very artificial, depending as it does upon a simple laboratory experiment, in connection with an obvious analysis of the general equations of motion. If this theory is to be preserved, then we must be assured that the atmosphere does in fact traverse the circuits prescribed, for it has been commonly assumed to conform with the facts of observation. As has been shown, observation does not bear out the requirements of the theory, and I shall, therefore, attempt to trace out in a descriptive way the circulation as it is developed over the North American Continent. The reduction of this kinematic picture to the corresponding mathematical form of dynamics is a task of very great difficulty, as may readily be inferred.

We may conceive the tropical strata to be elevated by thermal expansion relative to the polar regions, so that there is a certain average gradient slope and corresponding west-east velocity which is in equilibrium with it in accordance with the usual equations. If the canal theory of circulation is in operation we have poleward gradients in the upper strata, and equatorward gradients in the lower strata. If, instead of maintaining this circulation, there is an escape of currents from the Tropics in the lower strata in a poleward direction, then the gradients of the canal theory diminish and become much more moderate in consequence of this release of tension. Every such leakage current from the Tropics causes a break or fault in the gradient, and this must be attended by a corresponding deflection of the eastward current, through the operation of the deflecting force at right angles to it, due to the earth's rotation. Referring to the scheme of self-regulation of the circulation by the rise and fall of the gradients, fig. 18,

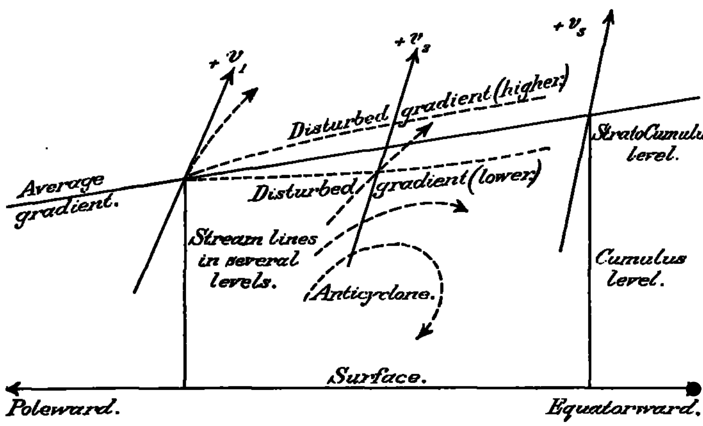

FIG. 18.—Scheme of self-regulation of the circulation by the rise and fall of the gradient.

we may consider the average gradient in the strato-cumulus level. When in its mean position, as determined by the whole set of natural circumstances which produce it, the isobars and the wind with a given direction and velocity are all practically in coincidence. The velocity is just enough to maintain the slope of pressure as measured by the gradient. If the thermal expansion of the Tropics increases, the gradient slope is elevated, the eastward velocity is increased, and this acceleration will continue to advance till an excess of energy secures for itself a way of escape. This increase of velocity is probably checked as follows:

From the Tropics in the lower levels, that is in the cumulus and strato-cumulus strata, a stream of air breaks away from the canal circuit, and pushes northward in some irregular course. This evidently causes a break in the gradient surfaces, such as is indicated, for example, by the dotted lines of fig. 18, and the slope of pressure which held the eastward drift in its position gives way. The action of the deflecting force due to the earth's rotation bends the current southward as is indicated, and there is thus made the beginning of an anticyclonic local movement. In the lowest levels, from cumulus down to the surface, the break in the average gradients may be so pronounced as to offer but little check to a complete anticyclonic gyration, such as appears at the surface of the earth. This deflection of direction causes a whirl and absorption of energy in the strata affected through the local interchange of inertia, and slows up the eastward drift by the vortex action which is produced with a downward component. The alternate rise and fall of the gradients, and the attendant anticyclonic gyrations, mark the successive efforts at self-regulation which the atmosphere as a thermal engine imposes upon itself. These continental pulsations are shown on the weather maps as the procession of high and low pressure areas which traverse the middle latitudes. In consequence of the superior velocity of eastward motion in the strato-cumulus level, this region is the first to feel the decline in gradient strength, due to the tendency of a stream to escape from the Tropics, as from the Gulf of Mexico over the United States. It thus happens that the anticyclone is not only larger in area, but it also generally precedes the cyclone in its formation. There are numerous instances in which the anticyclone overspreads the United States, while there is no important cyclone in connection with it, except possibly some depression or irregular action along the edges of the high area. Usually, however, incipient cyclones increase in intensity from these small beginnings, and they may even seem to drink up and exhaust the air which is flowing in anticyclonic circulation. This is the reason why I have heretofore described the anticyclone as preceding the cyclone in efficiency, and have thus reversed the order of action as taught by Professor Ferrel in his well-known theory.

The warm stream from the south is deterred from mingling with the cold anticyclonic air by the difference of its temperature, and the result is that the eastern side of the anticyclone, which flows southward, and the warm escape current from the south, flowing northward, compose two counter currents. These two currents together generate the cyclone, which is a vortex with an ascending central velocity. The gyration is produced by the action of the two independent streams acting like a couple, since they each depend upon separate gradient systems for their own mechanical pressures. It is not a gyration due to frictional impact, but rather to the steady pressure on the arms of a couple, since the streams are driven by independent gradients, and are held apart by having different temperatures in the two separate currents. The air is thus raised from the lower to the higher strata in great masses, by circulating through the configuration of the cyclone, and this, too, produces a retardation of the eastward drift by an immediate interchange in the inertia, since there is a quick mingling of air having different directions and velocities. Examples of this action can be seen by studying the Charts 20 to 35, inclusive, of the International Cloud Report. Furthermore, the motions of the atmosphere result in placing strata of air having different temperatures together, side by side, so that the surface isotherms are directed from the southwest to the northeast, and in consequence the northwest portions of a cyclone are cold and the southeast portions are warm. This does not conform to the requirements of either the Ferrel or the German theories, which demand a warm local center for the generation of cyclones. Yet if two such masses of air lay alongside while they are of different temperatures, an interchange of heat contents will take place locally between them, and thus the streams will interflow in such a way as to strengthen the cyclonic gyration. This will be accompanied by distinct stratification of the air currents in the local cyclones, such as is observed in the kite and balloon ascensions, since the air of different temperatures is drawn out into thin ribbons having large discontinuous surfaces, which are favorable to the interchange of heat. It is frequently found that a great anticyclonic area as it approaches the ocean, although attended by no cyclone, will yet suddenly cause a violent whirl to form on its edge by the mere action of these adjacent masses of different temperatures. Such local storms are sometimes formed on the Atlantic coast during a single night, and they may cause vortices with hurricane velocities on the coast. The line of junction of these warm and cold currents, along the southern and southeastern parts of the low area, is the locus of the formation of the majority of the tornadoes of the United States, the counter flow setting up the gyration, which is converted into a genuine columnar vortex, through which the heated air of the lower strata escapes into the colder strata of the higher levels. The hurricanes of the West Indies similarly form along the places of the counter flow between the Atlantic high area and the southeast trade winds when at their extreme northern limit, as in August to October. The vortex then travels westward and skirts around the periphery of the high area until it is absorbed finally in the eastward drift of the higher latitudes. Such dynamic intermingling of the general and the local circulation is, therefore, not only in accordance with observations, but it is a suitable substitute for the defective canal theory of the general circulation, and also for the untenable theory of the local cyclones and anticyclones, supposed to be dependent upon the central heat produced by condensation of the aqueous vapor of precipitation. This view is attended, on the other hand, by the following disadvantage: That while the canal theory and the warm center cyclone theory lend themselves readily to mathematical treatment and to analytic solutions of considerable elegance, we are obliged to substitute for them an irregular system of stream lines in the lower strata, not at all readily put into mathematical forms. This turbulent circulation, with its self-adjusting government of the eastward flow, its interaction between the general and the local vortices, its numerous subordinate phenomena, such as tornadoes, hurricanes, and cyclones, is easy to comprehend, but hard to analyze mathematically into the exact dynamic forces of equilibrium. It is possible to construct several special typical configurations for each district of the earth, as was done for the United States in Charts 20–35, inclusive, of the International Cloud Report, and then draw the stream lines, with their velocities, in order to prepare for the computation of the dynamic forces involved. This is the true meteorological problem of the future.

THE STRUCTURE OF THE ANTICYCLONE.

We will now examine a little more closely the structure of the anticyclone and the cyclone as given by observations for the sake of the analytical problems presented by their configuration. It has been claimed by meteorologists that there is a southward component in the middle strata of the north temperate zone toward the high pressure belt, but a northward component in the upper strata and another northward component near the surface, as is indicated on fig. 13, Paper IV[2]. The observations of 1896–97 do not, however, give such a distribution of the mean components, for they show that there is a very small average drift northward in all strata, increasing slightly with the height above the surface. That is to say, the atmosphere in the eastern and central United States drifts northward a very little and thus supplies part of the air that descends into the anticyclones through the upper strata. We have indicated how the leakage in the lower strata from the Tropics in part replaces the air which descends in the anticyclonic areas, and it is assumed that the small residual northward drift complements the amount that is required to fill the anticyclonic areas. The downward vortex, therefore, draws in a portion of the air passing through it from the upper strata, as a consequence of the gyration induced through the countercurrent action, and therefore the feeble northward component of the circulation of the higher strata seeks the surface practically in the middle latitudes, before arriving at the polar zone, through the mechanism of the local vortices. Hence, it follows that there is little cause for the formation of a general anticyclone close to the pole itself, which Ferrel assumed to exist; the result of Kimball's discussion in the MONTHLY WEATHER REVIEW, September, 1901, goes to show that this movement only feebly exists, and is in conformity with this exposition of the general circulation. Therefore, the air that descends in a local anticyclone comes from two sources, the leakage currents from the Tropics in the lower and middle strata and the feeble northward drift in all strata, especially the higher.

There is one feature of the anticyclonic vortex which may be mentioned, though it belongs more properly to an analytic treatment of that circulation. The anticyclonic components of fig. 6, Paper III[3], show that we are not dealing with a pure form of vortex. The two possible laws are typically the parabolic $\frac{v}{\varpi}=$ constant, and the hyperbolic $v\varpi=$ constant.

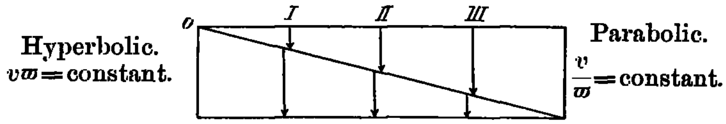

FIG. 19.—Mixed system of hyperbolic and parabolic components.

According to the parabolic law $\frac{v}{\varpi}=$ constant, the circulation causes simply a depression in the center of the gyrating fluid; according to the hyperbolic law $v\varpi=$ constant, there is a vertical component of circulation as in ordinary vortices. In the observed anticyclonic components the velocities v are about equal to each other on the I, II, III, circles, and it seems to me that this can only happen if there is a mixture of these two laws of motion. Thus we may divide the observed components v_{I}, v_{II}, v_{III}, into two parts by a diagonal as shown on fig. 19. The upper components represent the parabolic, and the lower the hyperbolic portions. This is physically necessary for the following reason. The anticyclone is formed by large currents of air moving in more or less independent streams on its outer portions, while only curling offshoots reach its central parts; this would produce the pure parabolic components only. But, through imperfect pressure gradients there is also near the center a true downward component of circulation, and this can be supplied only by the action of hyperbolic components, that is to say, of a simple vortex motion. Hence, the general anticlockwise movement of the anticyclone, strongest on the outer circles, has accompanying it a true downward or vortex component which lengthens the components u in the central portions. If this is correct, one sees an additional reason for holding that Ferrel's explanation of the anticyclone is impracticable, and also that the reversing of the cyclone to make an anticyclone, as proposed by Oberbeck and Pockels is not warranted. We need very accurate observations to settle so difficult a point of pure theory, but I can not at present see any other satisfactory explanation of the gyratory components derived from the Weather Bureau observations.

STRUCTURE OF THE CYCLONE.

In Table 18 we give the results of cloud observations in the United States.

In the following example the relations which should exist in a pure vortex are deduced for comparison with the data given under low areas in Table 18.

Taking the inward radial velocity $u=-1.25$, at the distance 1,250 kilometers, assuming $\lambda=.000100$ for $\theta=46°\,17'$, and $c=.000002$, also $k=.000050$, their introduction into the several formulæ gives the values found in Table 19.

An account will be given of the derivation of the formulæ in Paper VI.

It is seen that the rotational velocity v is about the same as that given by the observations up to the circle whose radius is 150 km = 93 miles from the center, as seen under v_2 of Table 18. The values of the radial velocity agree fairly well, if we admit that the observations may be somewhat imperfect for this component up to the region of the inner circle I, whose radius is 250 km. There the component u is much larger than expected in the upper strata, and this indicates some opposition to the free development of the vortex near the core. It is presumed that this implies a struggle to intrude into the rapid eastward drift, accompanied by a broadening of the vor-

[2] MONTHLY WEATHER REVIEW, April, 1902, Vol. XXX, p. 166.

[3] MONTHLY WEATHER REVIEW, March, 1902, Vol. XXX, p. 117.

TABLE 18.—*Anticyclonic and cyclonic velocities at each 1,000-meter level.* (*Copy of Table 126 International Cloud Report.*)

Cloud forms.	Height in meters.	High areas. I u_2	High areas. I v_2	High areas. II u_2	High areas. II v_2	High areas. III u_2	High areas. III v_2	Low areas. I u_2	Low areas. I v_2	Low areas. II u_2	Low areas. II v_2	Low areas. III u_2	Low areas. III v_2	Velocities in the general cyclone. u_1	Velocities in the general cyclone. v_1
Ci. and Ci. St.	10000	—3.5	—4.0	+ 4.5	—5.5	+2.5	— 3.0	—6.5	+ 3.0	—1.0	+ 3.5	0.0	—3.0	—2.8	+35.4
	9000	—3.5	—5.0	+ 4.5	—5.0	+2.5	— 8.0	—3.0	+ 8.0	—3.0	+11.0	—1.5	—1.0	—2.6	+35.0
Ci. Cu.	8000	—3.0	—6.0	+ 3.5	—4.5	+2.0	— 9.0	—2.5	+11.0	—5.5	+13.5	—2.0	+1.0	—2.4	+34.6
	7000	—1.5	—6.5	0.0	—4.5	—1.5	— 8.0	—6.5	+13.5	—5.0	+15.0	—2.0	+2.0	—2.2	+30.0
A. St.	6000	0.0	—7.0	— 2.5	—4.5	—4.0	— 7.5	—9.0	+15.0	—2.0	+15.5	—0.5	+3.0	—2.0	+25.0
	5000	0.0	—7.5	+ 2.5	—6.0	—2.5	— 8.0	—9.0	+17.0	0.0	+15.5	+1.5	+5.0	—1.8	+23.6
A. Cu	4000	0.0	—7.5	+ 8.5	—8.0	0.0	—10.0	—7.5	+20.0	0.0	+14.5	+2.5	+7.0	—1.6	+22.6
	3000	0.0	—7.0	+10.0	—9.5	+2.0	—12.0	—3.5	+23.0	0.0	+13.0	+2.0	+7.5	—1.3	+21.0
S. Cu.	2000	+1.0	—6.0	+ 7.5	—9.5	+2.0	—11.0	—1.0	+20.0	0.0	+11.0	0.0	+5.0	—1.0	+14.0
Cu. and St.	1000	+3.5	—4.5	+ 5.0	—7.5	+1.5	— 7.0	0.0	+ 8.0	—1.5	+ 8.0	—1.5	+4.0	—0.8	+ 6.4
Wind.	0000	+4.0	—2.5	+ 2.5	—4.0	+1.0	0.0	0.0	+ 6.0	—3.5	+ 3.0	—2.0	+3.0	—0.5	+ 1.3
Means of the velocities, u_2, v_2		—0.3	—5.8	+ 4.2	—6.2	+0.5	— 7.6	—4.4	+13.1	—2.0	+11.2	—0.3	+3.8		
Radius, ϖ			1		3		5		1		3		5		
Product, ϖv_2			—5.8		—18.6		—38.0		+13.1		+33.6		+19.0		

$+u_2$ = outward on radius.
$+v_2$ = anticlockwise about center.

I. $\varpi_{I} =$ 250,000.
II. $\varpi_{II} =$ 750,000.
III. $\varpi_{III} =$ 1,250,000.

$+u_1$ = south.
$+v_1$ = east.

TABLE 19.—*Application of the formulæ for a cyclone.* (*Copy of Table 127, International Cloud Report.*)

CONSTANTS AND FORMULÆ.

$\theta = 46° 17'$ $\lambda = 2n \cos\theta = .000100$

$u = -1.25$ $c = -\frac{2u}{\varpi} = .000002$ $\frac{\lambda}{k-c}\frac{c}{2} = .000002$

$\varpi = 1,250,000$ meters $k = .000050$

$\psi = -\frac{c}{2}\varpi^2 z.$ $u = -\frac{c}{2}\varpi.$ $v = +\frac{\lambda}{k-c}\frac{c}{2}\varpi z.$ $w = +cz.$ $\cot i = \frac{\lambda}{k-c}z.$

DERIVED DISTANCES, VELOCITIES, CHECKS, AND INCLINATIONS.

ϖ meters.	miles.	z	ϖz	u	v	w	ϖv	$\varpi w = -2uz$	cot i	i °	G mm.
1250000	777	1.00	1250000	—1.25	2.50	.0000020	3125000	2.5	2.00	26.6	0.34
1000000	621	1.56	1560000	—1.0	3.125	31	3125000	3.1	3.12	17.6	0.40
750000	466	2.78	2085000	—0.8	4.17	56	3125000	4.2	5.56	10.2	0.55
500000	311	6.25	3125000	—0.5	6.25	125	3125000	6.3	12.50	4.6	0.78
250000	155	25.00	6250000	—0.25	12.50	500	3125000	12.5	50.00	1.2	2.03
200000	124	39.06	7812000	—0.20	15.61	781	3125000	15.6	78.12	0.7	3.00
150000	93	69.40	10412000	—0.15	20.82	1388	8125000	20.8	138.80	0.4	5.37
100000	62	156.30	15630000	—0.10	31.26	3126	3125000	31.3	312.60	0.2	13.90
50000	31	625.00	31250000	—0.05	62.50	12500	3125000	62.5			
40000	25	977.00	39080000	—0.04	78.16	19540	3125000	78.2			
30000	19	1736.00	52080000	—0.03	104.16	34720	3125000	104.2			
20000	12	3906.00	78120000	—0.02	156.24	78120	3125000	156.2			
10000	6	15625.00	156250000	—0.005	312.50	.0312500	3125000	312.5			

tex tube through the resistance, v being smaller than it should be and u greater, making the angle of the inclination i much greater than it ought to be in pure vortex motion. As the computation of the vortex goes to the height of 10 miles, far beyond the altitude to which the ordinary cyclonic motion penetrates, this being only 3 or 4 miles in the moderate movements of the air, it is seen that v should theoretically attain enormous velocities very near the center. The difference between these and such as are actually observed may be regarded as measuring the energy expended in breaking up the cyclone in the higher levels, which can be balanced only by retarding the movements of the general cyclone. The vertical velocity w is extraordinarily small from the bottom to the top, and in a measure justifies the method of discussing the motions of the air in the cyclone as a case of horizontal movement. It is impossible theoretically, however, that such a cyclonic motion without a vertical component should exist at all. The fact is that a slow vertical movement is acting over the very large area covered by the cyclone, and is sufficient to carry off all the air which flows into it through a thin disk at the outside in horizontal directions toward the center. The checks ϖv and $\varpi w = -2\,u\,z$ hold throughout the cyclone, thus proving that our data are at least approximately correct. The angle i, between the tangential direction and the current, theoretically becomes very small within 300 miles of the center. It was shown by the results of the observations that the movement at the cumulus level is much more rounded than in the lower strata, the difference being caused by the retardation of the air operating upon the surface irregularities of the ground. A congested or irregular inflow near the center will similarly increase the angle i, since the component u is increased and v diminished by it. The observations given on the weather maps do not record the conditions within 60 miles of the center with any definiteness. In hurricanes the core is about 30 miles broad and its boundary is quite sharp, which shows that the component v is highly developed, while u is small. But the hurricane also penetrates to much greater altitudes, as already mentioned. Finally, the gradient shows that there is a slow change near the outer limit, but that it increases very rapidly on approaching close to the center.

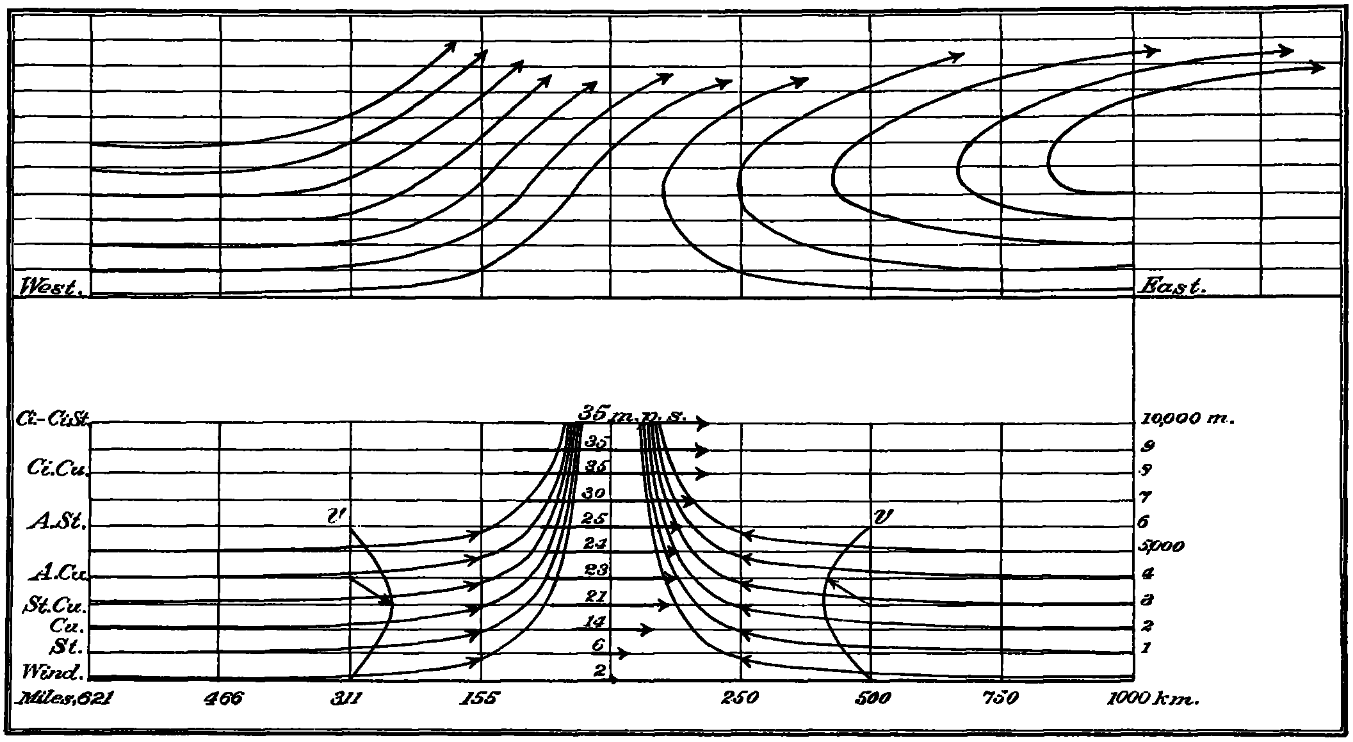

FIG. 20.—General scheme of the structure of cyclones.

THE SPECIAL FEATURES OF THE CIRCULATION.

The special features of the circulation indicated on fig. 20, which may be mentioned are as follows: (1) It is evident that this scheme avoids entirely the primary difficulties attending the Ferrel and also the German types of circulation. Each of these divided the cyclone into two parts having special properties. Ferrel divided his cyclone at a vanishing rotational velocity, $v = 0$, which involved a circulation in opposite directions on either side of it; the German type consists of two parts, separated by a discontinuous movement at the circle of the maximum velocity for v. The pure vortex, on the other hand, has only one law to deal with, and that, too, the simplest of all, in accordance with which the motion is generated. (2) Neither of the other types provides for a true calm region at the center of the cyclone, commonly observed in hurricanes as the eye of the storm. Ferrel's formula 402b shows that v increases from the circle $R = 0.707$ to the very center; the formulæ 450 and 471 show that the velocity v decreases gradually from the circle bounding the inner region toward the center, but does not vanish till reaching it. The construction here proposed indicates that since v is a function of the height z as well as of the radial distance ϖ, the air in streaming toward the center is gradually lifted above the ground by purely dynamic action and leaves a core without gyratory circulation. (3) The approach of a moving particle to the axis of the cyclone is attended by an increase of the velocity of rotation, which accelerates rapidly as it passes into the upper strata. There it accomplishes the work of deflecting the eastward drift, and it expends some of its energy in that way. The result of this opposition to free motion is to spread out the top of the vortex, reduce its gyratory velocity, and change the relations of v_2 and u_2. In the undisturbed gyratory motion the component v_2 becomes very great in comparison with u_2, and u_2 is always a small quantity. An inspection of Table 18 shows, however, that in the strata, between 5,000 and 10,000 meters the radial velocity u_2 is relatively large, and the angle of the inclination instead of being nearly 0° is from 25° to 35° on the inner circle II. This means that the original coefficient upon which the dimensions of the cyclone depend, namely, $c = -\frac{2u}{\varpi}$, does not remain a constant, but increases from the boundary toward the axis of the gyration. Thus we obtain,

$$c_{\mathrm{I}} = .0000352 = \frac{0.6}{1250000}$$

$$c_{\mathrm{II}} = .0000053 = \frac{4.0}{750000}$$

$$c_{\mathrm{III}} = .0000005 = \frac{8.8}{250000}$$

This involves an expenditure of energy in the struggle attendant upon intruding into the swiftly moving upper stratum. Furthermore, as was previously pointed out, Ferrel's theory of the slowing down of the excessive eastward velocities which would arise from the pure vortex law of the

TABLE 20.—*Dimensions and velocities in the waterspout off Cottage City, Vineyard Sound, Mass., August 19, 1896.* (*Copy of Table 128 International Cloud Report.*)

Working formulæ: $v^2=2gz$; $u=\frac{v}{z}$; $w=\frac{2v}{\varpi}$; $\varpi v=\text{Const.}$; $2zu=\varpi w$; $\frac{c}{z}=\frac{4g}{v\varpi}$.

Dimensions in meters.			Velocities in meters per second.				Dimensions in feet.			Velocity in miles per hour.		
h	z	ϖ	u	v	w	$2zu=\varpi w$	h	z	ϖ	u	v	w
1280	0	∞	—	0	0	0	4200	0	∞	—	0	0
1278	2	518.4	3.13	6.26	0.02	12.53	4193	7	1701	7.0	14.1	0.04
1189	91	76.9	0.46	42.24	1.10	84.49	3901	299	253	1.0	94.4	2.5
1097	183	60.8	0.29	53.39	1.76	106.79	3599	601	200	0.6	119.5	3.9
1006	274	44.3	0.27	73.31	3.31	146.61	3301	899	145	0.6	164.	7.4
914	366	38.3	0.23	84.72	4.42	169.45	2999	1201	125	0.5	189.	9.9
731	549	31.3	0.19	103.77	6.63	207.53	2398	1802	102	0.4	233.	14.9
549	731	27.1	0.16	119.73	8.83	239.47	1802	2308	89	0.4	268.	19.8
457	823	25.5	0.15	127.04	9.94	254.10	1409	2701	84	0.3	284.	22.2
366	914	24.3	0.15	133.89	11.04	267.78	1201	2999	79	0.3	300.	24.7
183	1097	22.1	0.13	146.68	13.25	293.36	601	3599	72	0.3	328.	29.6
146	1134	21.8	0.13	149.13	13.70	298.26	479	3721	72	0.3	333.	29.7
0	1280	20.5	0.13	158.44	15.46	316.89	0	4200	67	0.3	354.	34.6

$c=0.01208$

conservation of areas applied to the general cyclone, is that friction is largely concerned with the operation. It seems, however, that a much more efficient cause of retardation is the interaction between these two types of motion, namely, the linear and the rotary, by which the lower strata thrust themselves into the higher. The effect is to enlarge the size of the vortex tube at the top by the resistance, deflect the eastward drift into sinuous curves, slow down the eastward velocity, and thus restrain the general cyclonic movement from excessive values. (4) The resultant of these component forces and velocities is to produce a circulation along the parallels of latitude which may be represented by the upper part of fig. 20. The upper clouds of the cirrus region precede the cyclone proper as forerunners of this type of circulation; the lower clouds follow in succession, till precipitation is produced at the elevation of 1,000 to 3,000 meters; in very rapid circulations the eye of the storm is fully developed; the clearing up is more abrupt on the westward than on the eastward side of the cyclone. (5) The progressive movement of storms is partly an effect of the cyclone covering an area of sufficient extent to be in different latitudes, so that variations in cos θ amount to something. But other circumstances are more important. By Table 33, Section IV, International Cloud Report, it is seen that the northward components for the group of areas which are covered by the currents of air from the south are greater than those from the north. That is to say, the movement in the streams from the Tropics is more rapid than that from the polar regions, and the result is to roll up the eastward side of the cyclone to the north more than the westward to the south. The cyclone tends to rotate along the front of a high area toward the north. At the same time its top is fastened by means of the circulation into the eastward drift at 4,000 meters elevation, and these two components make the storm move northeastward in the central and eastern portions of the United States. If the general cyclone has other directions, as when hurricanes form in the Caribbean Sea, the same principles hold, and the storm there first moves westward, then northward and northeastward, because the general circulation is controlled in that region by the anticyclone in the southern portions of the north Atlantic Ocean. (6) It has been shown that the currents which feed the cyclone have different velocities at different altitudes, being greatest from 2,000 to 5,000 meters above the ground. Each stratum forms a stream for itself conforming to the general law, but modifies its dimensions according to the constants pertaining to the special locality. Since these different strata have thus distinct local movements, especially considering the variable temperatures and densities of the currents from the north and south, respectively, it follows that the conditions are favorable for the formation of turbulent minor circulations of all kinds. The movement of the air is therefore partly congested, and partly runs in free whirls, the difference of equilibrium in temperature being gradually reduced to the proper normal value for the latitude and altitude by this forced intermingling of the subordinate parts of the cyclone. This process of restoring to an equilibrium the temperature of masses, bearing with them that of the region from which they came, is generally completed by the time the 5,000-meter level is reached, judging from the records of the balloon observations. In summer, when the eastward drift is relatively slow, the pure vertical convective ascension may extend up to 10,000 meters. This is much more likely to happen on the eastward than on the westward side of a cyclone, because the vortex components throw back the eastward movement upon itself, and thus make the strata more stagnant in vertical directions. It has also been found practically very difficult to make the kites fly on the east of the low center, the best ascensions being made on the southerly and westerly quadrants. (7) The entire problem of analyzing the movements of the air in their details is so exceedingly complicated that only slow improvements in dynamic meteorology can be expected. A clearer idea of the fundamental conditions may, however, enable us to advance more rapidly than is now expected. It will be a very important gain if meteorology can free itself from some of the theories which have so long prevailed, but which now are seen to be quite untenable, and have seriously retarded its advancement.

THE VELOCITIES IN TORNADOES.

The motions in tornadoes are similar to those in cyclones, yet the tube is not only inverted in position, but the stream lines occupy only the central portions, and the lines of ψ become tangent to the plane whose height is H_0 at certain distances from the center. The fundamental equations for the tornado are 303–308. $\psi = +\frac{c}{2} \varpi^2 z$ holds for the tornado with vertical axis positive downward, and the single bounding plane at the distance H_0 above the ground. $\psi = -\frac{c}{2} \varpi^2 z$ is the equation for the cyclone with the vertical axis positive upward. A multitude of minor relations and comparisons can be drawn from the two sets of equations 303–308 and 488–490. If the lower part of fig. 20 be looked at as if the horizontal axis were the vertical, we have a picture of the tornado tube. The diagram is not good because not drawn to scale, but the idea is easily understood. Hence, one law serves for all types of local storms, cyclones, hurricanes, and tornadoes, which thus theoretically differ from each other only in their dimensions and in the details by which they are formed. Cyclones are generated chiefly by horizontal convection currents; hurricanes have a stronger vertical convection current and also horizontal convection currents; tornadoes arise chiefly from vertical convection, assisted by some horizontal currents which counter flow in the cumulus level.

It may be remarked that the stream lines indicated by Ferrel, page 300, Recent Advances, are conjectural only, and do not conform to the theory of stream lines in a vertical vortex tube, nor to observation, which shows that the air is quiet close up to the boundary of the tornado tube.

THE WATERSPOUT OFF COTTAGE CITY, MASS., AUGUST 19, 1896.

The result of the computation on this interesting waterspout is added, and it shows the dimensions and velocities in metric and English measures which were derived from the observed distances and the formulæ. The most important feature is the value of the vertical velocity of 35 miles per hour at the sea level. See Table 20.

ANNALS OF THE ASTROPHYSICAL OBSERVATORY OF THE SMITHSONIAN INSTITUTION, VOLUME I.[1]

By S. P. LANGLEY, Smithsonian Institution, dated April 29, 1902.

The work upon the infra-red solar spectrum, described in this volume, is the latest outcome of investigations with the bolometer, begun with the invention of that instrument by the writer about the year 1880. In the use of the bolometer (in principle so well known now as to need no description) the practise at Allegheny during the nearly ten years that studies of the infra-red spectrum of the sun were in progress there, required at least two and preferably three observers. One made and read the settings of the spectroscope, a second read as often as he could the indications of the galvanometer, while the third recorded all the observations and operated the shutter in front of the slit of the spectroscope. In those days the galvanometer, though far less sensitive than that now employed, was rarely free from "drift," or wandering of the spot of light upon the scale and this "drift" was usually so rapid that the spot of light would cross the whole scale within a few minutes.

It had been my intention at Allegheny to replace the tedious and incomplete system of eye observations at the galvanometer by an automatic photographic recording apparatus, but the "drift" stood as a seemingly insuperable obstacle in the way. The difficulty was attacked immediately after the installation of apparatus at the Astrophysical Observatory at Washington in 1891. Without stopping to mention details, it need only be said that by persistent efforts carried on through more than ten years as described in the early chapters of the Annals, and with the aid of several devices which the reader may find there illustrated, these obstacles have been removed and it has become possible to use the galvanometer with perfect success to produce a purely automatic record, in connection with the bolometer, and with a photographic recording apparatus.[2]

As thus perfected, the procedure of mapping a portion of the spectrum is this:

A prismatic spectrum is formed by a spectroscope of the so-called "fixed arm type," in which, as the reader will recall, the ray which is in minimum deviation is given a certain fixed direction by an optical method, so that the observing instrument remains fixed also. This observing instrument is here the bolometer, whose essential part is a Wheatstone's bridge, chiefly formed of two excessively narrow and thin strips of platinum, only one of which is exposed to the radiation to be studied. On looking into the eyepiece of the bolometer, this appears like a single spider web wire of a micrometer, seen against the background of the spectrum, for instance just below the *A* line, whose position might be read by it on the large azimuth circle on which the bolometer is mounted. But this thread is not alone a fiducial index, for since it conveys an electric current it is also sensitive and comparable to a nerve which will recognize the heat or cold which falls on it, and not only recognize but accurately measure it by the changes set up in the current in question.

The whole forms an electrical thermometer which actually measures to very much less than the millionth of a degree.

As the bolometer strip is warmed or cooled the galvanometer mirror turns to and fro, and the spot of light which it reflects passes to and fro on its scale, which is here a photographic plate. A clockwork of extreme accuracy moves the rock salt prism so that the spectrum marches uniformly across the bolometer thread at a rate, let us say, of one minute of arc in one minute of time, while the photographic plate moves as regularly in a vertical direction at right angles to the movements of the galvanometer spot at a rate, for example, of a centimeter in one minute of time. There is thus produced a curve called a bolograph, in which ordinates accurately correspond to relative amounts of energy, and abscissae to deviations in the prismatic spectrum. Depressions in this curve correspond to cooler lines or bands in the spectrum; the visible Fraunhofer lines, which are cool to the bolometer, appear as such deflections in the bolographs; the invisible lines, which are wholly insensible to the eye and chiefly insensible to the photographic plate, but which are recognized everywhere by the bolometer and discovered to fill the whole infra-red, are recorded also.

The first object of the principal research described in the Annals was to map out in detail the hitherto scarcely explored region of the solar spectrum between the limit of the visible, just below *A* at about 0.8μ and a wave length of 5.3μ, beyond which little energy reaches the earth from the sun. In the accompanying illustration, Plate I (similar to Plate XX of the Annals), two bolographs taken on different days are superposed to show the coincidence of their indications, which directly represent the solar energy in this (invisible) part of the spectrum, and beneath is a corresponding line spectrum drawn to show the region mapped in a more conventional aspect. At the left we see the comparative space occupied by the Newtonian or visible spectrum, on the scale of the average dispersion of the prism in the infra-red. Above is a curve obtained by Lamansky about thirty years ago, which gives the sum of the knowledge of the infra-red spectrum at that time, and was then justly regarded as a great triumph.

In the work of the Astrophysical Observatory over 700 lines

[1] Washington, D. C., Government Printing Office, 1900.

[2] The "drift," long the great enemy to bolometric research, has become so much reduced that often it amounts to less than a single centimeter an hour, whereas in the old days a centimeter a minute would have been regarded as relative perfection.